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Time-Domain Analysis of Lumped-Distributed Networks

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Abstract—**A new method for time-domain analysis of networks containing transmission lines and lumped linear/nonlinear elements is presented. A key feature of the method is a procedure for generating a system matrix in a manner that involves only sums of subnetwork (or element) terms (no products or quotients). Numerical integration algorithms are used to reduce the problem to a solution of sparse algebraic equations.**

I. Introduction

TIME-DOMAIN analysis of lumped-element networks is well established. Powerful analytical and numerical techniques are readily available, including the popular state-space and Laplace transform methods. General purpose computer programs such as SCEPTRE [1] and SPICE [2] provide easy-to-implement time-domain solutions for complex lumped systems even when nonlinear, time-varying, and/or active elements are included.

The development of methods for transient analysis of mixed lumped-distributed networks is of relatively recent origin, and general techniques that permit, for example, lossy transmission lines of arbitrary lengths and nonlinear active lumped elements are not yet available. Yet, the time-domain analysis of such networks is increasingly important in design considerations of fast switching digital integrated circuits, broad-band radar and communication systems, time-domain reflectometry systems, and in the study of lightning and EMP effects in systems containing transmission lines, to mention only a few applications. The purpose of this paper is to present a technique suitable for the analysis of a very general class of lumped-distributed networks.

During the course of this study, a substantial literature search was carried out. The most pertinent articles and books are listed for the reader's convenience [3]–[17]. While the technique to be presented is significantly different from the methods found in the literature, the present concept grew from a "wouldn't it be nice if..." thought session following a May 21, 1976, reading of Silverberg's [3] paper. Since that time, the new technique has been successfully applied to a wide variety of problems. The impact of Silverberg's work is gratefully acknowledged.

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II. System Equation Formulation: Part I

Consider systems which have network models consisting of interconnections of linear distributed elements (e.g., TEM transmission lines, waveguides), lumped linear or nonlinear elements, dependent sources, and independent sources. Partition the network into two parts as shown in Fig. 1. One part consists of linear (distributed and/or lumped) elements. The other part contains any lumped nonlinear or time-varying elements and independent sources.

Silverberg's [3] procedure is to solve for the terminal behavior of the linear part of the network in the frequency domain and then convert to a terminal time-domain description by numerical inverse-transform techniques. The time-domain solution for the whole network is obtained step-by-step in time at the interface of the two parts, by a simultaneous solution of a convolution equation representing the linear part with a differential equation representing the nonlinear part. The simultaneous solution is accomplished at each time increment by solving algebraic equations obtained by application of the trapezoidal integration rule to the original equations.

For the moment let us focus our attention on the linear part of the network. Wouldn't it be nice if the frequency-

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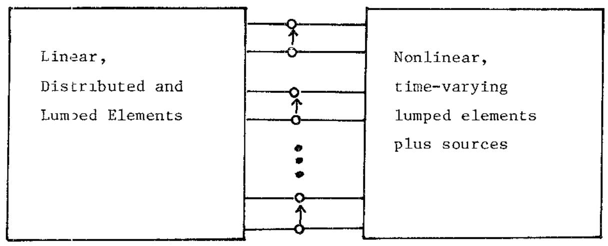


Fig. 1. Partitioned network.

domain calculations and the inverse-transform calculations could be eliminated and all calculations be performed directly in the time domain? Computer program complexity, memory requirements, and computational time could all be significantly reduced. The catch is that we would need a way of combining element descriptions to form network descriptions such that the resulting network matrix is directly compatible with convolution solutions. Basically this implies that the overall system matrix should contain only sums or differences of individual element responses (no products or quotients allowed). The indefinite admittance matrix [18] appeared a good possibility, but because of the type of systems to be considered, a port-description method rather than a terminal-description method was desired. Kron's transformation methods [19] provided the inspiration for the technique to be described. However, formal transformation techniques turn out to be unnecessary, a very simple algorithm sufficing. At this point, the problem statement for the linear part of the network is the following. Determine a scheme for representing networks, such that given the terminal step response of the subnetworks (or elements) in the time domain, the time-domain terminal step response for the connected overall network can be determined as sums and differences of the individual subnetwork responses. Then, by convolution the time-domain terminal response of the overall network can be determined for any specified set of inputs.

A. Combining Subnetworks

Short-circuit admittance parameters will be used (a dual-impedance representation has also been used successfully). The underlying feature of the method is to treat every kind of connection as though it is a parallel connection. This approach requires the addition of open-circuited ports in certain situations. Such additional ports are like ideal voltmeter connections enabling determination of voltage at that point in the network without disturbing the system. The added open-circuited ports increase the size of the system matrix but the associated current is zero and the overall system matrix is sparse. The net effect has thus far appeared to be an increase in computational efficiency.

As a first example consider the cascade connection of the two 2-ports as shown in Fig. 2(a). Common practice would have us multiply the individual *ABCD* parameters to obtain the new *ABCD* parameters for the cascade

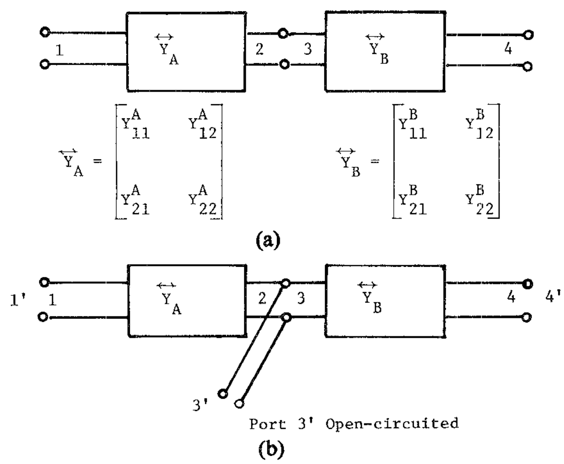


Fig. 2. (a) Cascade connection of two 2-ports. (b) Cascade connection treated as a parallel connection with added open-circuit port.

connection. However, we are now constrained to use Y parameters and to avoid products and quotients of individual terms in our overall description. This can be accomplished as follows. Notice that the cascade connection of Fig. 2(a) to form a new 2-port can be treated as a parallel combination of ports 2 and 3 to form a new 3-port as shown in Fig. 2(b). If port 3′ is open-circuited, then physically the networks of Fig. 2(a) and 2(b) are identical. However, the mathematical descriptions are different. In the first case the resulting network is treated as a 2-port, while in the second case it is treated as a 3-port with $I_3'=0$. The resulting y matrix for the cascade connection treated as a constrained 3-port is determined as follows. First form the Y matrix for the unconnected subnetworks.

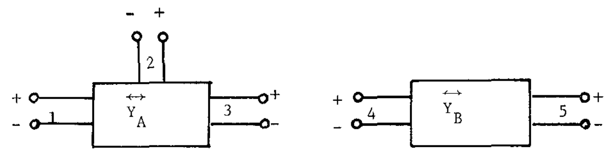
$$\overleftrightarrow{Y}_{\mathrm{SUB}}=\begin{array}{c|c|c|c|c|} & 1 & 2 & 3 & 4 \\ \hline 1 & Y_{11}^A & Y_{12}^A & 0 & 0 \\ \hline 2 & Y_{21}^A & Y_{22}^A & 0 & 0 \\ \hline 3 & 0 & 0 & Y_{11}^B & Y_{12}^B \\ \hline 4 & 0 & 0 & Y_{21}^B & Y_{22}^B \\ \hline \end{array}\ . \qquad (1)$$

The rows and columns of $\overleftrightarrow{Y}_{\mathrm{SUB}}$ corresponding to ports to be connected in parallel are now added. Ports 2 and 3 combine to form port 3′, while ports 1 and 4 become 1′ and 4′, respectively. The result is the desired Y matrix for the cascade combination treated as a constrained 3-port.

$$\begin{array}{l} \overleftrightarrow{Y}_{\mathrm{CASCADE}}= \\ I_3'=0 \end{array}\begin{array}{c|c|c|c|} & 1' & 3' & 4' \\ \hline 1' & Y_{11}^A & Y_{12}^A & 0 \\ \hline 3' & Y_{21}^A & Y_{22}^A+Y_{12}^B & Y_{12}^B \\ \hline 4' & 0 & Y_{21}^B & Y_{22}^B \\ \hline \end{array}\ . \qquad (2)$$

This representation of a cascade connection involves only *sums* of the subnetwork element admittances. The more conventional 2-by-2 matrix representation for the cascade connection can be obtained by eliminating V_3' from the

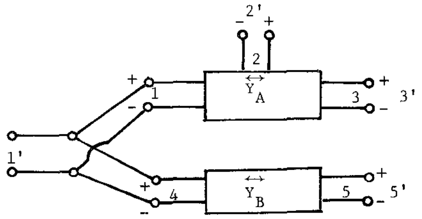
(a)



$\overleftrightarrow{Y}_{SUB}$ =

	1	2	3	4	5
1	Y^A_{11}	Y^A_{12}	Y^A_{13}	0	0
2	Y^A_{21}	Y^A_{22}	Y^A_{23}	0	0
3	Y^A_{31}	Y^A_{32}	Y^A_{33}	0	0
4	0	0	0	Y^B_{11}	Y^B_{12}
5	0	0	0	Y^B_{21}	Y^B_{22}

(b)



$\overleftrightarrow{Y}'$ =

	1'	2'	3'	5'
1'	$Y^A_{11}+Y^B_{11}$	Y^A_{12}	Y^A_{13}	Y^B_{12}
2'	Y^A_{21}	Y^A_{22}	Y^A_{23}	0
3'	Y^A_{31}	Y^A_{32}	Y^A_{33}	0
5'	Y^B_{21}	0	0	Y^B_{22}

Fig. 3. (a) Unconnected subnetworks. (b) Ports 1 and 4 connected in parallel yielding a new 4-port network. Rows and columns 1, 4, of Y_{SUB} are added to obtain Y.

system equations (since $I_3'=0$). The resulting Y matrix is

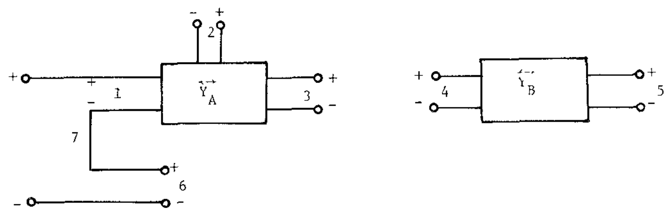
$$\overleftrightarrow{Y}_{\substack{\text{STANDARD}\\ \text{CASCADE}}} = \begin{array}{c|c|c} & 1' & 4' \\ \hline 1' & Y^A_{11} - \dfrac{Y^a_{21}Y^a_{22}}{Y^A_{22}+Y^B_{11}} & \dfrac{-Y^B_{12}Y^A_{12}}{Y^a_{22}+Y^B_{11}} \\ \hline 4' & \dfrac{-Y^A_{21}Y^B_{21}}{Y^a_{22}+Y^B_{11}} & Y^A_{22} - \dfrac{Y^B_{21}Y^A_{12}}{Y^a_{22}+Y^B_{11}} \end{array} \tag{3}$$

which obviously includes products and quotients of individual 2-port terms, thereby complicating a solution by convolution.

True parallel connections are simple and require no added open-circuited ports. A parallel connection of one port of a 3-port network with one port of a 2-port network to form a new 4-port network is illustrated in Fig. 3.

A series interconnection of ports in terms of admittance parameters under the constraint that only sums of individual subnetwork admittance parameters appear in the result requires a little more ingenuity. An auxiliary connecting network is introduced. The series connection of a pair of ports is illustrated in Fig. 4 using the networks of Fig. 3(a). Port 1 of network A is to be series connected to port 4 of network B. Series T is connected to port 1 and the Y matrix modified as shown. This operation is easily done automatically by a computer upon receiving the command for a series connection.

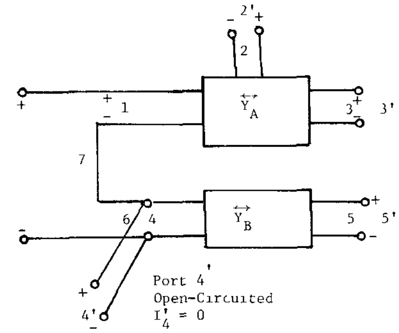
(a)



$\overleftrightarrow{Y}_{sub}$ =

	7	2	3	6	4	5
7	Y^A_{11}	Y^A_{12}	Y^A_{13}	$-Y^A_{11}$	0	0
2	Y^A_{21}	Y^A_{22}	Y^A_{23}	$-Y^A_{21}$	0	0
3	Y^A_{31}	Y^A_{32}	Y^A_{33}	$-Y^A_{31}$	0	0
6	$-Y^A_{11}$	$-Y^A_{21}$	$-Y^A_{31}$	Y^A_{11}	0	0
4	0	0	0	0	Y^B_{11}	Y^B_{12}
5	0	0	0	0	Y^B_{21}	Y^B_{22}

(b)



$\overleftrightarrow{Y}'$ = $I_4' = 0$

	1'	2'	3'	4'	5'
1'	Y^A_{11}	Y^A_{12}	Y^A_{13}	$-Y^A_{11}$	0
2'	Y^A_{21}	Y^A_{22}	Y^A_{23}	$-Y^A_{21}$	0
3'	Y^A_{31}	Y^A_{32}	Y^A_{33}	$-Y^A_{31}$	0
4'	$-Y^A_{11}$	$-Y^A_{21}$	$-Y^A_{31}$	$Y^A_{11}+Y^B_{11}$	Y^B_{12}
5'	0	0	0	Y^B_{21}	Y^B_{22}

Fig. 4. (a) Unconnected subnetworks with "A" modified for series connection of port 1. (b) Series interconnection of port 1 of "A" with port 4 of "B".

The series, parallel, and cascade connections of pairs of ports, permit very general networks to be configured from subnetworks (or elements). The method presented above permits system equations to be formulated involving only sums of subnetwork admittance terms as desired.

III. System Equation Formulation: Part II

Return now to the total network consisting of linear distributed and lumped elements plus nonlinear and time-varying lumped elements. The network is partitioned as shown in Fig. 1. The solution procedure is as follows. First, the short-circuit step-response matrix for the linear part of the system is established as sums of the individual subnetwork terms as described in the preceding section, then a matrix convolution equation is formed relating port voltages and currents at the interface between the linear and nonlinear network parts. The interface port voltages and currents are simultaneously constrained by the equations for the nonlinear part of the network. Both convolution and nonlinear equations are represented numerically by using trapezoidal (or another appropriate technique) integration leading to a set of simultaneous algebraic equations relating port voltages and currents at each time increment. Solution of these equations yields desired voltages and currents at each time increment.

The step-response matrix for each linear network may be determined by measurement or calculation. Let $\overleftrightarrow{A}(s)$ be the step-response matrix in the Laplace transform domain and $\overleftrightarrow{Y}(s)$ be the short-circuit admittance matrix. Then,

$$\overleftrightarrow{A}(s)=(1/s)\overleftrightarrow{Y}(s) \tag{4}$$

$$\overleftrightarrow{a}(t)=\mathcal{L}^{-1}\{\overleftrightarrow{A}(s)\}=\mathcal{L}^{-1}\{(1/s)\overleftrightarrow{Y}(s)\} \tag{5}$$

where $\overleftrightarrow{a}(t)$ is the step-response matrix in the time domain.

Interface port currents and voltages are constrained by the linear part of the network as follows, where $\vec{I}(s)$ and $\vec{V}(s)$ are vectors of port currents and voltages, respectively:

$$\begin{aligned}\vec{I}(s)&=\overleftrightarrow{Y}(s)\vec{V}(s)\\&=\left[\frac{1}{s}\overleftrightarrow{Y}(s)\right]\left[s\vec{V}(s)\right]\\ \vec{i}(t)&=\mathcal{L}^{-1}\{\vec{I}(s)\}\end{aligned} \tag{6}$$

or

$$\vec{i}(t)=\overleftrightarrow{a}(t)*\dot{\vec{v}}(t)U_{-1}(t)+\overleftrightarrow{a}(t)\vec{v}(0^+) \tag{7}$$

where $\dot{\vec{v}}(t)$ is the time derivative of the port voltage vector, $\vec{v}(0^+)$ is the initial value of the port voltage vector, $U_{-1}(t)$ is a unit step, and $*$ implies convolution. The nature of the nonlinear elements is assumed to be such that a description of the form

$$\dot{\vec{v}}(t)=\overleftrightarrow{f}(\vec{v}(t),\vec{i}(t),t) \tag{8}$$

is possible, where $\overleftrightarrow{f}(\vec{v}(t),\vec{i}(t),t)$ is a matrix whose elements are explicit functions of $\vec{v}(t)$, $\vec{i}(t)$, and t. Equations (7) and (8) describe the network completely. Given the initial conditions on $\vec{v}(t)$, we can in principle solve for $\vec{v}(t)$ and $\vec{i}(t)$ from (7) and (8). Unless the matrix of functions is extremely simple, the solution must be obtained numerically. Any implicit integration technique may be used. Trapezoidal (fixed or variable interval) and Gear-type algorithms [20] have proven very satisfactory. For ease of presentation the fixed interval trapezoidal method will be presented.

Let Δ be the interval between time points. Then, (7) can be written as

$$\begin{aligned}\vec{i}(k\Delta)=\frac{1}{2}\sum_{j=1}^{k}&\left[\overleftrightarrow{a}([k+1-j]\Delta)+\overleftrightarrow{a}([k-j]\Delta)\right]\\&\cdot\left[\vec{v}(j\Delta)-\vec{v}([j-1]\Delta)\right]+\overleftrightarrow{a}(k\Delta)\vec{v}(0^+),\\&k=1,2,\cdots.\end{aligned} \tag{9}$$

In each increment, the step response is approximated by the average of its two end-point values and the derivative of the voltage is approximated by the divided difference of its end-point values. Notice that $\vec{i}(k\Delta)$ in (9) can be separated into two parts, one depending on the past history and the other on the current value of $\vec{v}(k\Delta)$, as follows:

$$\vec{i}(k\Delta)=\vec{i}_0(k\Delta)+\overleftrightarrow{g}_0\vec{v}(k\Delta),\qquad k=1,2,\cdots \tag{10}$$

where

$$\overleftrightarrow{g}_0=1/2\left[\overleftrightarrow{a}(\Delta)+\overleftrightarrow{a}(0^+)\right] \tag{11}$$

$$\begin{aligned}\vec{i}_0(k\Delta)=\frac{1}{2}\sum_{j=1}^{k-1}&\left[\overleftrightarrow{a}([k+1-j]\Delta)+\overleftrightarrow{a}([k-j]\Delta)\right]\\&\cdot\left[\vec{v}(j\Delta)-v([j-1]\Delta)\right]\\&-\overleftrightarrow{g}_0\vec{v}([k-1]\Delta)+\overleftrightarrow{a}(k\Delta)\vec{v}(0^+).\end{aligned} \tag{12}$$

Thus $\overleftrightarrow{g}_0$ is a constant matrix equal to the average step response during the first time interval. The vector $\vec{i}_0(k\Delta)$ can be treated as a set of current sources whose values are determined by the past history of $\vec{v}(k\Delta)$. For a given value of k, $\vec{i}_0(k\Delta)$ is known. In effect, a lumped time-varying terminal equivalent circuit has been obtained for the linear (lumped-distributed) part of the overall network.

For the nonlinear part, from (8) we have

$$\vec{v}(t)=\int_{t-\Delta}^{t}\overleftrightarrow{f}(\vec{v}(\tau),\vec{i}(\tau),\tau)d\tau+\vec{v}(t-\Delta). \tag{13}$$

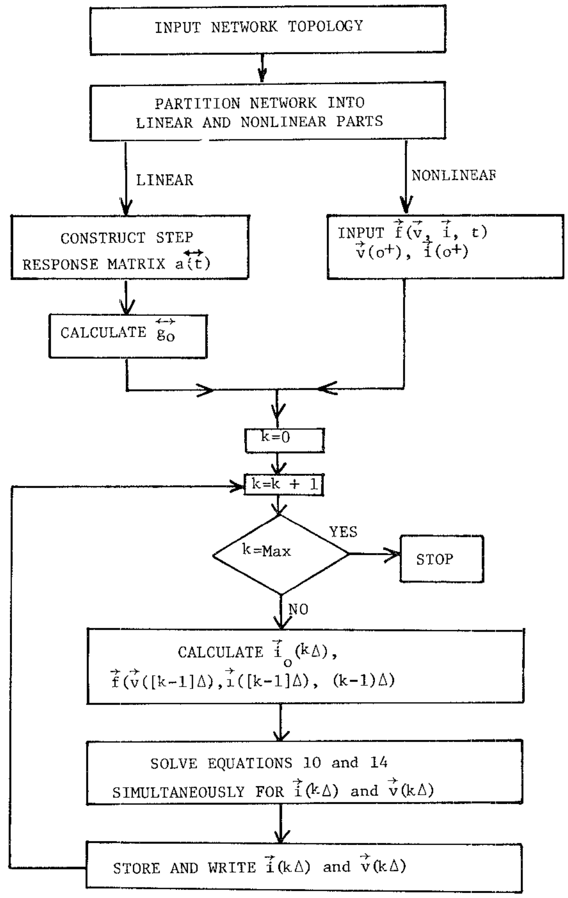


Fig. 5. Simplified flow chart for solution procedure.

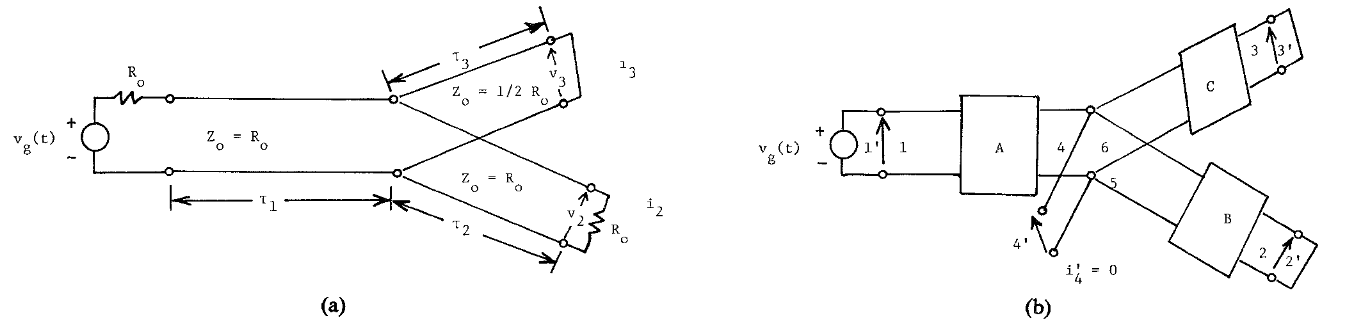


Fig. 6. Circuit for *Example 1.* $R_0=50\ \Omega$, $\tau_1=\tau_2=\tau_3=10\ \mu s$, $v_g(t)=U_{-1}(t)$ V.

By the trapezoidal integration rule we have

$$\vec{v}(k\Delta)=\frac{\Delta}{2}\vec{f}(\vec{v}(k\Delta),\vec{i}(k\Delta),k\Delta)+\frac{\Delta}{2}\vec{f}(\vec{v}([k-1]\Delta),\vec{i}([k-1]\Delta),(k-1)\Delta)+\vec{v}([k-1]\Delta), \quad \text{for } k=1,2,\cdots \tag{14}$$

where $\vec{v}(k\Delta)$ is separated into two parts, one depending on the current value and the other on the past history of $\vec{v}$ and $\vec{i}$.

The solution for the overall network is obtained by solving for $\vec{v}(k\Delta)$ and $\vec{i}(k\Delta)$ simultaneously from (10) and (14) at each time increment $k=1,2,\cdots$. Note that the system of equations is algebraic even when the network contains distributed elements. The simplified flow chart of Fig. 5 summarizes the solution procedure.

IV. Examples

The following examples were chosen such that they could be verified by hand calculations, and to clearly detail the solution procedure.

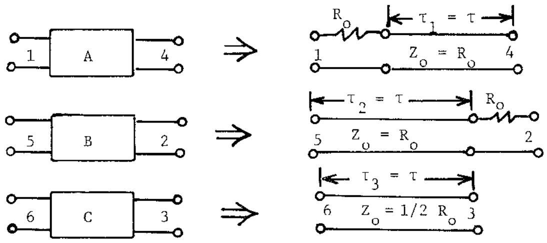
Example 1: Three lossless transmission lines are interconnected as shown in Fig. 6(a). Determine the currents i_1, i_2, and i_3, given that $v_g(t)=U_{-1}(t)$ V.

The network is first redrawn, breaking the circuit into subnetworks whose step responses are known, and adding open-circuited ports at any required points as shown in Fig. 6(b). Let $\Delta=1\ \mu s$. This information is supplied to the computer causing the unconnected subnetwork matrix to be established as given in (15) (zeros are not stored).

$\vec{a}_{SUB}(t)=$ (15)

	1	4	5	2	6	3
1	a_{11}^A	a_{12}^A	0	0	0	0
4	a_{21}^A	a_{22}^A	0	0	0	0
5	0	0	a_{11}^B	a_{12}^B	0	0
2	0	0	a_{21}^B	a_{22}^B	0	0
6	0	0	0	0	a_{11}^C	a_{12}^C
3	0	0	0	0	a_{21}^C	a_{22}^C

The a_{ij}'s for these subnetworks are given in Fig. 7. A



(a)

$$a_{11}^A=a_{22}^B=\frac{1}{2R_o}\left[U_{-1}(t)+U_{-1}(t-2\tau)\right]$$

$$a_{12}^A=a_{21}^A=a_{12}^B=a_{21}^B=-\frac{1}{R_o}U_{-1}(t-\tau)$$

$$a_{22}^A=a_{11}^B=\frac{1}{R_o}U_{-1}(t)$$

$$a_{11}^C=a_{22}^C=\frac{1}{R_o}\left[U_{-1}(t)+2U_{-1}(t-2\tau)+2U_{-1}(t-4\tau)+\dots\right]$$

$$a_{12}^C=a_{21}^C=\frac{-4}{R_o}\left[U_{-1}(t-\tau)+U_{-1}(t-3\tau)+\dots\right]$$

(b)

Fig. 7. (a) Subnetwork designations. (b) Step-response elements for the subnetworks of (a) with $\tau_1=\tau_2=\tau_3=\tau$.

variety of subnetwork terms frequently needed are stored and available in the program. New a_{ij}'s may be input as equations, tables, or measured data.

Next the interconnection information is input which, in this case, causes rows and columns 4, 5, and 6 to be added yielding the connected network matrix given in (16). The individual a_{ij}'s are given in Fig. 7(b).

$\vec{a}(t)=$ (16)

	1	2	3	4
1	a_{11}^A	0	0	a_{12}^A
2	0	a_{22}^B	0	a_{21}^B
3	0	0	a_{22}^C	a_{21}^C
4	a_{21}^A	a_{12}^B	a_{12}^C	$a_{22}^A+a_{11}^B+a_{11}^C$

Notice that the primes have been dropped from the port designations to simplify writing the equations. Interface

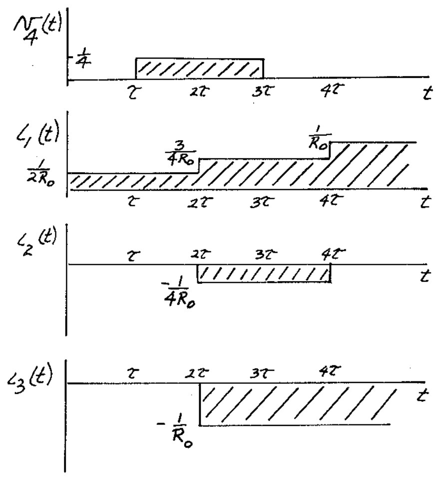


Fig. 8. Solution for *Example 1*, volts and amperes.

constraints are now imposed. In general this would involve a set of equations representing the nonlinear and source part of the system. In this example, the constraints are simply $v_1 = U_{-1}(t)$, $v_2 = 0$, $v_3 = 0$, and $i_4 = 0$. Initial conditions are $v_1(0^+) = 1$, $v_2(0^+) = v_3(0^+) = v_4(0^+) = 0$.

Equation (10) can now be written for this example as

$$\begin{bmatrix} i_1(k\Delta) \\ i_2(k\Delta) \\ i_3(k\Delta) \\ 0 \end{bmatrix} = \begin{bmatrix} i_{01}(k\Delta) \\ i_{02}(k\Delta) \\ i_{03}(k\Delta) \\ i_{04}(k\Delta) \end{bmatrix} + \begin{bmatrix} \frac{1}{2R_0} & 0 & 0 & 0 \\ 0 & \frac{1}{2R_0} & 0 & 0 \\ 0 & 0 & \frac{2}{R_0} & 0 \\ 0 & 0 & 0 & \frac{4}{R_0} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ v_4(k\Delta) \end{bmatrix} \tag{17}$$

where i_{01}, i_{02}, i_{03}, and i_{04} must be calculated at each time increment using (12).

For $k=1$, $i_{01}(\Delta) = i_{02}(\Delta) = i_{03}(\Delta) = i_{04}(\Delta) = 0$, so that from (17) we obtain

$$\begin{aligned} i_1(\Delta) &= 1/2R_0 \\ i_2(\Delta) &= 0 \\ i_3(\Delta) &= 0 \\ v_4(\Delta) &= 0. \end{aligned} \tag{18}$$

No change occurs in any variable until $k\Delta = \tau$. For $k=10$, i.e., $k\Delta = \tau$, $i_{01}(10\Delta) = i_{02}(10\Delta) = i_{03}(10\Delta) = 0$, and $i_{04}(10\Delta) = -1/R_0$, so that (17) now yields

$$\begin{aligned} i_1(10\Delta) &= 1/2R_0 \\ i_2(10\Delta) &= 0 \\ i_3(10\Delta) &= 0 \\ v_4(10\Delta) &= 1/4. \end{aligned} \tag{19}$$

No further change occurs until $k\Delta = 2\tau$, at which time i_1, i_2, and i_3 all change. The solution proceeds as indicated with the final results shown in Fig. 8.

Example 2: Let the network of *Example* 1 be modified to include a nonlinear element as shown in Fig. 9. The input voltage is now $v_g(t) = tU_{-1}(t)$, for $t \leqslant 2$ μs, and $v_g(t) = 0$, for $t > 2$ μs. All other parameters for *Example* 1 remain unchanged. Determine $v_2(t)$ and $v_3(t)$.

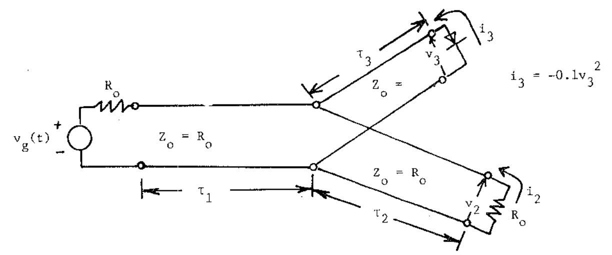


Fig. 9. Circuit for *Example 2.* $R_0 = 50$ Ω, $\tau_1 = \tau_2 = \tau_3 = 10$ μs, $v_g(t) = tU_{-1}(t)$, for $t \leqslant 2$ μs, $=0$, for $t > 2$ μs.

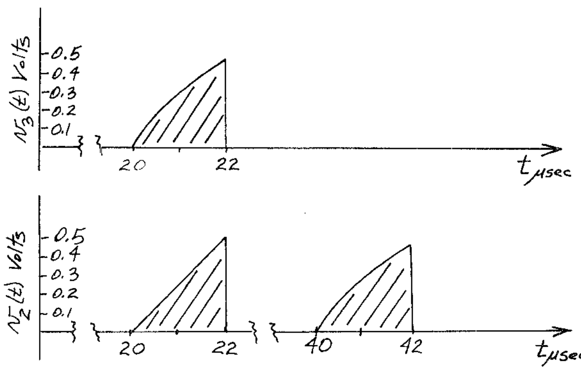


Fig. 10. Solution for *Example 2*, volts.

The setup for *Example* 1 remains unchanged except for the new input voltage and the constraint imposed on the output port of block "C" by the nonlinear device. With $\Delta = 0.5$ μs and the proper nonlinear constraint imposed, the program yields the results of Fig. 10, which can be easily verified by hand calculations.

Example 3: A network consisting of three sections of lossy but distortionless transmission lines with RC loads, as shown in Fig. 11, is driven by a step current generator.

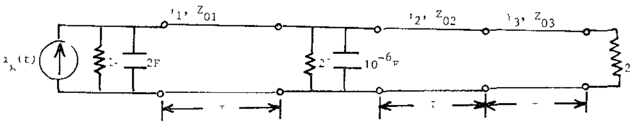


Fig. 11. Circuit for *Example 3.* $i_g(t)=2U_{-1}(t)$ A, $\tau=0.5$ s, $\gamma_1=\gamma_2=\gamma s+1$, $Z_{01}=1\ \Omega$, $Z_{02}=Z_{03}=2\ \Omega$.

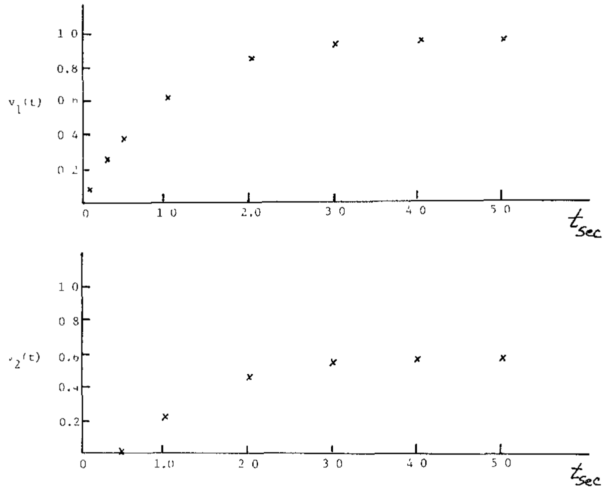


Fig. 12. Solution for *Example 3*, volts.

Determine $v_1(t)$ and $v_2(t)$. This example is one used by Silverberg [3]. The exact solutions for this problem are

$$v_1(t)=(1-e^{-t})\ \mathrm{V}$$

$$v_2(t)=(e^{-0.5}-e^{-t})U_{-1}(t-0.5)\ \mathrm{V}. \tag{20}$$

Results computed by the computer program are shown in Fig. 12 and agree with the exact results to six decimal places.

V. Conclusion

A new method for time-domain analysis of lumped-distributed networks has been presented. The method is easy to implement and has been successfully applied to a number of networks involving transmission lines and lumped linear and nonlinear elements. Tutorial examples are provided to illustrate the working details of the method.

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